# Units of Angles

James R. Clynch
February 2006

I. Degrees Minutes Seconds of Arc

There are several units that are commonly used for angles. In addition, there is confusion because latitude and longitude are usually expressed as three numbers, the first being the number of whole degrees, the second the whole number of minutes and the third the number of seconds. Here minutes and second really mean minutes of arc and seconds of arc.
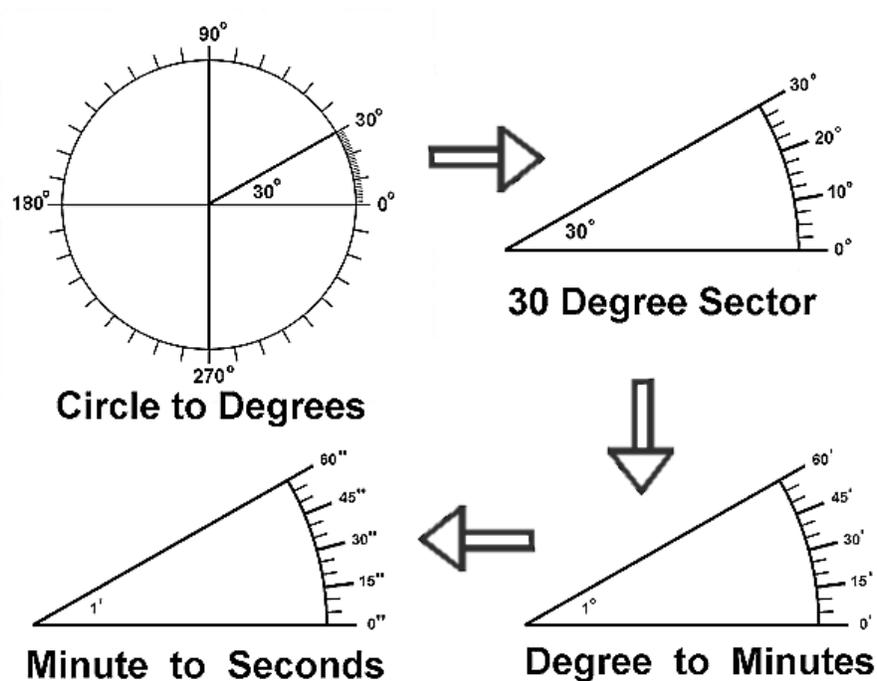
Angle Units Relationships

As can be seen in the drawing, the each smaller unit is 1/60 th of the last unit. For example, 1 degree is equal to 60 minutes and 1 minutes is equal to 60 seconds. Often minutes of arc is denoted by a single quote (') after the number and seconds of arc by a double quote (").

An example is the location of the Naval Postgraduate School:

> 36 35' 42.2" N
> 121 52 28.6 W

which in degrees is

> 36.5950528 N
> 121.8746273 W

Note: in science work, east longitudes are normally used. The longitude of NPS is therefore -121.8746273 E or 238.1253727 E.

Be careful to not confuse these angle units with time units. To make things more confusing, there is an angular unit of hours - used in astronomy, but it is not the same as a degree. In fact 24 hours of angle is a circle, making 1 hour if arc 15 degrees.

II. Radians

In addition to these "human oriented" units, there is a "natural" unit for angles.   This is called the radian and is closely related to the length along the arc of a circle.
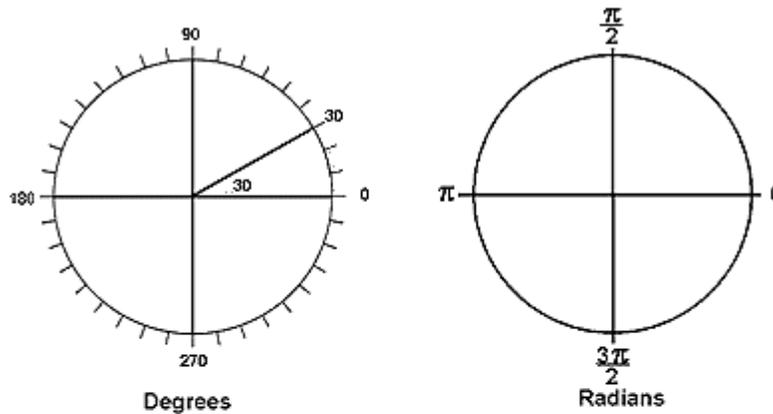
360 degrees	=	2π radians
180 degrees	=	π radians
57.29... deg	=	1 radian
1 deg	=	0.01745... radians

Degrees   Radians

Natural Angular Units - Radians

There are 2π radians in a circle (360 degrees), π radians in 180 degrees, and $\pi/2$ radians in a right angle (90 degrees). Radians are the units of angle needed to compute trigonometric functions like sine and cosine. Many calculators accept degrees as input

and convert to radians to compute the function. Most computer languages require the programmer to pass an angle in radians.

As can be seen, radians don't usually come out to nice even values. This is because the value of pi is an irrational number. This means that it does not have an exact representation. The value is a never ending number. To 5 decimal places :

$$pi = 3.14156$$

but it really goes on forever. In many books of mathematics tables values of pi to a very large number of digits is given. For example the *CRC Mathematical Tables* gives

pi = 3.14159 26535 89793 23846 26433 83279 ....

The value of radians as angular units commonly occurs in two places, in computer programs and in finding arc lengths.